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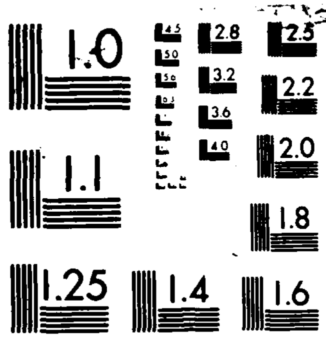

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AN ANALYTICAL FRAMEWORK FOR EFFICIENCY EVALUATION AND DETERMINATION OF THE PREFERRED MAIN BATTLE TANK FLEET

THESIS

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Captain, USA

AFIT/GOR/OS/86D-13

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Operations Research

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Preface

The purpose of this research was to develop an analytical framework for the efficiency evaluation and the determination of the preferred main battle tank fleet. In support of the research effort, the United States Army Armor School has provided the data which formed the basis for the derivation of an effectiveness measure for United States armor units. The derived effectiveness measure describes the output capability of an armor unit to attrit threat forces in combat. After the capability of each United States armor unit is assessed the unit's efficiency in producing tank kills can be determined relative to all units that comprise the United States armor fleet. When the relative unit efficiency is known, critical decisions regarding the best use and employment of tank assets can be determined through sensitivity analysis of the effectiveness measure.

The only limitation that affects the results presented in this thesis concerns the derivation of the attrition rate coefficients for the unit effectiveness measure. The attrition rate coefficients needed for this research to be complete will be obtained upon subsequent assignment to the Army Concepts Analysis Agency. The attrition rates used in this study facilitate the illustration and the utility of the methodology presented.

I am extremely indebted to my faculty advisor, Major Daniel W. Reyen, USA, for his assistance and guidance throughout the course of this research effort. His comments,

concern, and supportive counseling in the writing and preparation of this thesis are greatly appreciated. Above all else, I wish to thank my wife Tina for her understanding and assistance in maintaining the supportive atmosphere necessary for the successful completion of this thesis research.

Michael S. Remias

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Abstract

The primary objective of this thesis research was to determined the relative unit efficiencies of the expected attrition capability of United States armor forces. A measure of effectiveness defining the unit attrition capability was derived using the Lanchester-type equations of heterogeneous combat as a functional basis. Included in the armor effectiveness measure was a parameter describing the unit's discounted time value to the attrition process. The discounted time value of a unit is characterized by exponential decay reflecting the situationally dependent value of the unit to influence the battle engagement.

The determination and analysis of the relative unit efficiencies was accomplished using the data envelopment model of Charnes, Cooper, and Rhodes. The non-linear relation characteristic of the efficiency equation reduces to a linear programming problem by the method of linear fractional programming. From this analysis and methodology, important decisions regarding the efficient deployment and employment of armor assets can be quantitatively assessed. The results of this thesis research indicate that the analytical methodology contained in this thesis can be used as a method for the comparison of armor procurement, production and employment options.

AN ANALYTICAL FRAMEWORK FOR EFFICIENCY EVALUATION AND DETERMINATION OF THE PREFERRED MAIN BATTLE TANK FLEET

I. Introduction

In recent years the acquisition of military equipment for the armed forces of the United States has come under increased scrutiny both by Congress and the general public. Reports of excessive costs in repair parts and defense industries not adhering to procurement contracts has effected a review of the acquisition process by all the services. On 3 July 1986 the Associated Press reported that a presidential committee had found several deficiencies in the readiness of United States forces. The committee stated that:

> Overall defense decision making by tho executive branch can be improved...combatant forces can be organized and commanded better...control and supervision of the entire acquisition system-including research, development, and procurement-can be strengthened and streamlined. (3:2)

The preceeding statement implies that measures must be taken to strengthen the national security of the United States and the readiness posture of the armed forces.

The combat readiness of United States forces is a complex mix of several diverse factors. The ability of the United States to effectively fight and win the next war depends on the efficient orchestration of men, materiel, and national resolve in a coordinated effort against the Threat.

To this end, it is of paramount importance that our armed forces overcome the deficiencies noted by the presidential committee. Particularly important is the statement made by the committee that the decision making process can be improved and the reorganization of combat forces could increase the effectiveness of our war fighting ability (3:2). The research question pursued in this thesis addresses the need for an analytical framework to assist the decision maker in determining the most efficient use of our military resources. The importance of this research effort can not be over-emphasized. In light of the committee's findings, it is clear that our national leadership has recognized that a problem exists in defense decision making within the acquisition process. It is also evident that our forces could be better organized into a more efficient and effective fighting force.

An important factor that bears on the problem of decision making within the executive branch of the government is the fact that the defense industry must operate within the limits of the defense budget. Difficult questions must be answered concerning where defense dollars will be allocated. The decision to commit limited resources of funds in an effort to increase the nation's defense posture must realize the maximum benefit for each dollar expended. The question must be asked; how can one measure a nation's "defense posture" to know if a decision to commit resources results in an increase in defense capability? What factors bear on the

problem of defining the "effectiveness" of a defense posture? How does one measure the "efficient" allocation of diverse and varied resources? The Department of Defense has a difficult task in measuring the performance of its product, that is, the defense of the nation. Quantitative methods to assist defense decision-makers and managers are necessary to justify requests for appropriations of defense dollars. The development of a methodology to assist the Army leadership in the determination of the most efficient use of its weapon system resources proposes to remedy, in part, some of the deficiencies and problems outlined above. The organization of the combat forces of the Army based on the most efficient allocation of combat resources can only serve to make the Army a more effective fighting force capable of winning the next war.

One of the most important elements of our land combat power is the United States armor force. Our military doctrine and strategy for land combat depends upon the effective employment of armor in battle as a maneuver force supported by combined arms operations. The United States Army Armor School is currently studying the question of how to allocate armor assets to provide for the most effective fighting capability in the combined arms battle. Two plans have been formulated in recent years in an attempt to answer this question. As the M1 main battle tank was being produced in the early 1970's, the M1 Main Battle Tank Production plan considered only the projected production rate of the new

tank. The question of the efficient integration of the new tank into the fleet was not addressed. Initial guidance from Department of the Army directed that the United States Army Europe would be the first major field command to receive the new tank along with the training units at Fort Knox, Kentucky and a test unit at Fort Hood, Texas. In 1983, the M1A1 Forward Fielding Plan attempted to define policy that directed the distribution of tank assets among units as the new tank increased its share in the total armor force (15:2). The focus of the M1A1 Forward Fielding Plan was to increase the combat effectiveness of units expected to meet the greatest potential threat, perceived to be the Soviet Union in the Central European theater. Two studies conducted by the United States Army Training and Doctrine Command in conjunction with the Armor School attempted to define and use a fleet effectiveness measure in an effort to determine the most preferred procurement and distribution option for armor resources. The Army Tank Program Analysis study was completed in 1983. The Armor Investment Strategy study was completed in 1985. An examination of some of the limitations uncovered in these two studies is necessary to understand the motivation for this thesis research.

Background

The Army Tank Program Analysis study analyzed the combat effectiveness of armor forces resulting from six initial procurement options identified by Department of the Army. The current measure of the combat effectiveness of United States

tanks is the weighted triple sum expressed by the following fleet effectiveness equation:

$$\text{FLEET LER} = \sum_{i=1}^{12}\sum_{j=1}^{7}\sum_{k=1}^{3}\left[(LERijk)*(Bi)*(Rj)*(Mk)\right] \qquad (1.1)$$

where: i = 1 if unit equipped with M60A1 or less
= 2 if unit equipped with M60A1(AT) or less
= 3 if unit equipped with M60A3
= 4 if unit equipped with M60A3(AT)
= 5 if unit equipped with M1
= 6 if unit equipped with M1(AT)
= 7 if unit equipped with M1+
= 8 if unit equipped with M1+(AT)
= 9 if unit equipped with M1E1/M1E2
= 10 if unit equipped with M1E1/M1E2(AT)
= 11 if unit equipped with M1E3(AT)
= 12 if unit equipped with FACS

where: j = 1 if unit equipped with T62 or less
= 2 if unit equipped with T62(90+) or less
= 3 if unit equipped with T64/T72
= 4 if unit equipped with T64(90+)/T72(90+)
= 5 if unit equipped with T80
= 6 if unit equipped with T80/(90+)
= 7 if unit equipped with FST(90+)

where: k = 1 if Blue is defending
= 2 if Blue is attacking
= 3 if Blue is delaying

LERijk = Loss exchange ratio of a Blue (United States) unit equipped with type tank (i) in mission profile (k) against a Red (Soviet) unit of type tank (j).

Bi = Percentage of Blue units equipped with type tank (i).

Rj = Percentage of Red units equipped with type tank (j).

Mk = Percentage of time Blue units are conducting mission (k).

AT = Advanced technology.

M1+ = M1 uparmored to M1E1 protection levels.

90+ = Threat advanced technology round.

(2:4-124).

The cumulative fleet effectiveness measure described above, equation (1.1), was used to evaluate the relative ability of the six procurement options to wage war. The input data for the cumulative fleet effectiveness equation was derived from analysis of the expected force ratios of opposing Blue and Red forces in the Central Europe environment. Percentages of Blue forces were obtained by determining the D-Day force and the D+90 force available with type (i) tank. The term "D-Day" refers to the actual day when the battle engagement begins. Calculations were made for each year beginning in year 1985 up to year 2000 for each procurement option. The percentage of Red units equipped with type (j) tank for each year was determined from analysis of the expected Threat force available at D-Day and at D+90. For the D+90 force examination all Red units expected to oppose United States forces in Central Europe were assumed to be used. The percentage of time that Blue units were conducting the missions of defend, attack, and delay were derived from classified requirement studies conducted by the Army Concepts Analysis Agency. As such, these values will not be presented here. However, they were determined from the results of combat simulation modeling conducted at the Concepts Analysis Agency. The loss exchange ratios used in the cumulative fleet effectiveness equation were derived from battalion level

combat simulation using the CARMONETTE land combat model (2:4-124).

A second study conducted by the Armor School resulted in the production of the Armor Investment Strategy in April of 1985. The objective of this study was to determine the impact of all vehicle production choices and improvements to the United States tank fleet for the period 1985 to the year 2000 The triple sum methodology using the loss exchange ratios derived from CARMONETTE modeling was again incorporated in this study with few minor changes. The cumulative fleet effectiveness equation (1.1) was modified to include only three types of Blue tanks. The United States tanks examined in this study included the categories of M60A3 and below, the M1 tank, and the M1A1 tank. The important aspect here is that the Army Investment Strategy study simpified the effort at calculating the cumulative fleet effectiveness by examining only three categories of Blue tanks. In addition, only three Soviet tanks were analyzed in the Threat force estimate. The three Red tank categories included in this analysis were the T72 and below, the T80, and the Future Soviet Tank (FST). Finally, the percentage of time that Blue units are conducting operational mission (k) incorporated only the two mission postures of defense and attack.

One of the important enhancements of the Army Investment Strategy study was the information provided concerning assessment of the composition of the Threat force in the area of interest (1:177). As the composition of the Red force

changed over time, the determination of the cumulative fleet effectiveness measure for United States tanks would also change. The Army Investment Strategy study included data depicting the Threat profile using actual numbers of tanks expected to engage United States forces in Central Europe. The numerical estimates were based on the analysis of both NATO and Warsaw Pact order of battle plans as these planned forces were processed through the DIA-CIA Land Armament and Manpower Model (1:178). The Threat numerical estimates in the Army Investment Strategy study are an important variable in the development of the methodolgy in chapter II of this thesis.

Four important limitations of the analysis conducted in the Army Tank Program Analysis and Army Investment Strategy studies are evident from the information provided in this review. The analysis of fleet effectiveness conducted in both studies used loss exchange ratios derived from battalion level combat simulation. Results from the CARMONETTE model were obtained through the analysis of the loss of forces in battalion level, thirty minute engagement iterations. The purpose of these studies addresses the fleet effectiveness of division level armor assets. As such, the research question in this report contends that a methodology to determine fleet effectiveness or fleet capability of division level armor assets should be derived from the analysis of at least division level land combat simulation. The two studies reviewed here also assumed that in those

instances where specific Blue-Red force engagements could not be modeled, valid loss exchange ratios could be extrapolated based upon the next best scenario run in CARMONETTE (2:4-135). The caveat upon this assumption made it clear that "professional military judgement was used to insure that the values were reasonably correct" (2:4-135). In effect, the modelers conducted what could be regarded as "Turing" tests (5:401) to insure the validity of the output results.

A second limitation of equation (1.1) concerns the attrition of engaged combat forces over time. Loss exchange ratios do not account for the time sensitive nature of the dynamics of close-in battle. They reflect a relative effectiveness ratio of opposing tank capabilities modeled in a thirty minute engagement simulation. One might expect, as is the contention in this thesis, that a more accurate measure of tank capability could be derived from analysis of division level engagements simulated over a period of days. A value for the attrition of opposing forces and the determination and use of attrition coefficients in place of loss exchange ratios proposes to lend greater insight into actual tank capability. The methodology portion of this thesis will address the issue of attrition modeling.

A third limitation characteristic of a loss exchange ratio is the fact that this measure is inherently non-linear in nature. The effort to determine an optimal solution involving non-linear relationships often involves very complex and comprehensive solution techniques such as convex

programming, quadratic programming, and solution of Kuhn-Tucker conditions. The methodology adopted in this thesis and the techniques used to determine fleet effectiveness and the efficient use of tank resources will eliminate the problem of non-linearity and make the task of solution easier for the analyst.

A fourth limitation of the fleet effectiveness methodology is uncovered through dimensional analysis of the component variables in the equation. If the ability of the armor force is measured in war fighting capability, the desired output from the Blue fleet effectiveness measure should be the number of Red tanks destroyed in combat. The following example demonstrates the incompatibility of the use of the percent of Blue tanks with the objective of determining a fleet effectiveness measure:

Scenario: An M60 tank force of 3000 tanks engages a T62 tank force of 7000 tanks. The Blue force is in an attack posture for 50 % of the engagement. The loss exchange ratio of T62 to M60 is 2:1. The total Blue fleet numbers 9000 tanks and the total Red fleet numbers 35000 tanks.

Dimensional analysis using equation (1.1) yields:

$$\frac{(2)\text{T62}}{(1)\text{M60}} \bullet \frac{(3000)\text{M60}}{(9000)(\text{Blue Tanks})} \bullet \frac{(7000)\text{T62}}{(35000)(\text{Red Tanks})} \bullet \frac{5}{10}$$

The equation reduces to:

$$\frac{(2)(\text{T62})(\text{T62})}{30(\text{Blue Tanks})(\text{Red Tanks})}$$

Now if it is assumed that T62 = Red Tanks, then the equation reduces to the following:

$$\frac{(2)\text{T62}}{(30)(\text{Blue Tanks})} \qquad (16).$$

The value determined above is a fractional exchange ratio of T62 to Blue tanks. The result does not measure the capability of the M60 to destroy the T62 because the desired output of the number of T62 tanks killed cannot be determined. The methodology adopted in chapter II will correct this apparent inconsistency in variable terms.

The current methodology of determining fleet effectiveness measures is a very time consuming process. There are at least six procurement options or plans outlined in the Army Tank Program Analysis and Army Investment Strategy studies that have been analyzed for their respective effectiveness measures. The generation of new options and hybrid options can be an easy process, especially when the upper echelons of Army leadership and the Congress want answers right now. A team of analysts using the present methodology could work an entire day to input the data, perform the effectiveness calculations, and conduct sensitivity analysis of the possible outcomes. The methodology and approach to the determination of unit and fleet efficiency measures contained in this thesis proposes to simplify the task of the analyst in producing a timely report.

Time is an important resource in terms of how it affects the mobilization of the fighting force. In the event that general war were to occur in Central Europe, the fast and efficient mobilization of the defense industry is necessary to allow the military to engage its full war fighting

potential. Difficult decisions concerning materiel procurement should be based on wartime contingencies. However the best contingency plans often fail to address an issue that surfaces after the line of departure has been crossed. The defense industry cannot rest during peacetime but must remain capable of reacting to wartime requirements. The mobilization of military resources must be done correctly, efficiently, and quickly during time of war. The methodology of determining unit and fleet efficiency measures will facilitate the decision to commit vital weapon resources during the mobilization process (16).

Problem Statement

In light of the limitations identified in the preceeding section, a particular need exists to develop and implement a methodology to determine the optimal fleet profile for United States armor forces. The methodology adopted must simultaneously address the questions of defining a fleet effectiveness measure and the efficient allocation of armor assets. The methodology adopted should be adaptable for use on a micro-computer to assist analysts in replicating and influencing efficiency determinations of future fleet distribution plans. The insights gained from research on the armor fleet should be applicable to the general question of the efficient use of any weapon system. The methodology that evolves from this thesis research should allow other analysts the ability to apply it to other efficiency problems as an aid to the decision-making process.

Research Objectives

Subject to the resource limitations outlined in the Army Tank Program Analysis and the Army Investment Strategy studies, the primary objective of this thesis research is to develop a methodology for determining the preferred Main Battle tank fleet for the United States Army.

The successful accomplishment of the primary objective of this thesis research will be based upon the completion of the following subsidiary objectives:

1. Combine the following resource characteristics into a mathematical relationship that describes a meaningful measure of the capability of armor forces:

    A. Unit parameters

        (1) Unit identification
        (2) Mission profile
        (3) Type of tank
        (4) Numbers of tanks

    B. Tank parameters (US)

        (1) Identification
        (2) Number on hand
        (3) Specified production schedule

    C. Tank parameters (Soviet)

        (1) Identification
        (2) Type of tank
        (3) Number on hand
        (4) Forecasted production schedule

    D. Attrition parameters

        (1) Attrition coefficients based upon combat mission profile for each type United States (Blue) and Soviet (Red) tank.

2. Based on the capability measure determined above, adopt an analytical framework within which

the efficiency of the distribution of armor assets can be determined.

3. Determine the relative unit efficiencies for all of the major field units considered in the Army Investment Strategy study.

4. Based upon the relative unit efficiencies, recommendations will be made on how to improve the overall fleet efficiency of the armor force.

Scope of the Research

The scope of the research will be confined to the data provided by the United States Army Armor School and the Army Concepts Analysis Agency. Specifically, the tank resource data contained in the Army Tank Program Analysis sample procurement option and distribution plan (2:B-I-3 to B-I-18) for United States tank assets will form the primary data base for the determination of an armor effectiveness measure. The mission profiles adopted in the Army Investment Strategy study (1:176) along with the Threat estimate of the Soviet fleet composition (1:179-181) will be incorporated for use in the derivation of an armor effectiveness measure. The attrition coefficients discussed in the chapter on methodology will be provided by the Army Concepts Analysis Agency. The thesis will not attempt to define the optimal tank type for either United States or Soviet tanks as this is not a weapon system design problem. The relative unit efficiencies for the major field units considered in this thesis will be determined for the time period of 1987 to the

year 2000. In addition, a cumulative fleet efficiency measure will be determined over this same time period.

II. Methodology

Conceptual Attrition Equation

The point of departure in developing a modeling approach that captures the effectiveness of a weapon system must begin with a basic understanding of the conceptual attrition equation. The problem of capturing the combat process of a division level engagement involves the aggregation of multiple weapon systems and processes such that the complexity of the real battle is often concealed within the attrition rate coefficient (11:1-17). Aggregated combat models represent the average results of many combatants acting out over a period of time by the rates at which various combat processes occur(11:1-17). The conceptual attrition equation that describes the basic interaction of engaged weapon systems over a period of time could be expressed as:

Y CASUALTIES = X FIRERS * ATTRITION RATE * DELTA T

where:

X FIRERS = Average number of friendly (Blue) shooters in the battle

ATTRITION RATE = Average rate at which a single friendly shooter kills an enemy (Red) system

DELTA T = Length of the engagement (expressed in terms consistent with the attrition rate)

(11:1-17)

Although this equation appears simple it is not trivial in nature because much of the dynamics of the battle, i.e. the

variables that effect the attrition process such as terrain, target acquisition, and battlefield environment, are captured in a single variable, attrition, whose meaning may be difficult to understand or compute (11:1-18). The development of an effectiveness measure in this chapter assumes the use of initial values for both the X and Y forces (19:38).

The concept of attrition in combat has been examined and modeled extensively using the Lanchester-type models of combat. These models were first formulated by F. W. Lanchester in his work, "Aircraft in Warfare: The Dawn of the Fourth Arm - No. V., The Principle of Concentration" (12). Lanchester type combat models are the principal variety of force-on-force analytical attrition models (19:35). For the purpose intended in this thesis, that is, the development of a methodology that relates combat effectiveness to efficiency it is imperative to begin the process by determining a measure of capability for the United States armor fleet. The modeling process begins with the basic conceptual design of the Lanchester attrition process (16). If one considers the modeling of a combat engagement between two homogeneous forces, then the force-on-force attrition process can be depicted as in figure 1 (18:9).

The fundamental paradigm of Lanchester combat theory assumes the casualty rate of a homogeneous force X is equal to the product of the single-weapon-system-type kill rate and the number of opposing firers Y.

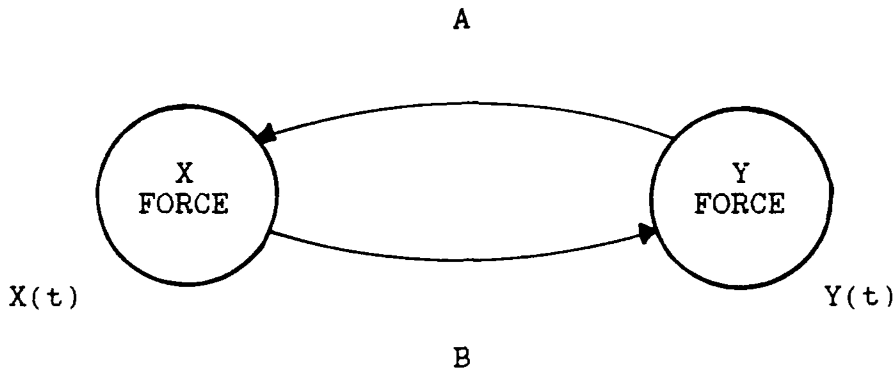

Figure 1. Homogeneous Force Combat

Combat between two homogeneous forces, as conceptualized by the basic Lanchester-type paradigm. The quantities A and B (here assumed to be constant) are called Lanchester attrition-rate coefficients. The coefficient A denotes the rate at which one Y firer kills X targets. Consequently, it represents the fire effectiveness of the weapon-system-type used by the Y force in the operational circumstances of the battle under consideration (18:9).

The relationship is defined by the following differential equations:

$$\frac{dx}{dt} = -Ay \quad \text{with} \quad x(0) = X \qquad (2.1)$$

and,

$$\frac{dy}{dt} = -Bx \quad \text{with} \quad y(0) = Y \qquad (2.2)$$

where

$x(t)$ = number of X firers at time (t)
$y(t)$ = number of Y firers at time (t)
t = 0 denotes time at which battle begins

The value of A is the rate at which a single Y firer kills X targets and is called a Lanchester attrition rate coefficient (19:38). Likewise, the value for B is the rate at which a single X firer kills Y targets. The values for the coefficients A and B are assumed to be positive and the preceeding minus sign denotes this value as "drawing down" the attrited force in question. For the purpose of this thesis, it is assumed that the value of A and the absolute value of -A are one and the same. Further, it is assumed that the values of X and Y forces at time t = 0 (when the battle begins) are positive. The fundamental assumption in support of the basic Lanchester equations (2.1 and 2.2) can be stated as:

(A1) The casualty rate of a force is equal to the product of the single-weapon-system-kill rate and the number of enemy firers (18:10).

The basic equation (2.1) assumes a constant value, A, for the single-weapon-system-type kill rate over time. In the

dynamics of the combat engagement, the effectiveness of a weapon system changes over time as the range between firer and target changes with time. Therefore, a time dependent attrition rate coefficient could be expressed as:

$$\frac{dx}{dt} = -A(t)y \qquad (2.3)$$

The basic attrition rate equation still holds if A = A(t) for any given point in time. Assumption (A1) remains valid at this same point in time. The enhanced Lanchester model (2.3) now relates the attrition of the X force with the capability of the Y force to attrit X (namely A), the number of Y shooters, and the time interval (t) over which the attrition process occurs.

The Lanchester equation (2.3) can be enriched by the assumption that the attrition of the X force is also dependent upon the number of X targets that are available to be killed in the combat engagement with the Y force. The enriched equation now can be expressed as:

$$\frac{dx}{dt} = -A(tx)y \qquad (2.4)$$

The Lanchester attrition rate coefficient now describes the weapon system performance of the Y force as dependent upon the time duration of the engagement (t) and the number of X targets available to kill and A = A(t,x) (18:12).

The total force casualty rate of equation (2.4) is still based upon assumption (A1). The attrition of the X force is still a function of the attrition rate A and the number of

firers Y. The basic equation (2.1) is now weighted by the enhancements of both the time duration of the combat engagement and the number of targets available to kill.

The "fully enriched" (18:13) basic Lanchester paradigm for homogeneous force combat incorporates a final element in the modeling process. If it assumed that the single-weapon-system-type kill rate A depends not only on time (t) and the number of targets X, but also the number of firers Y, then the basic Lanchester equation in enhanced form can be expressed as:

$$\frac{dx}{dt} = -A(t,x,y)y \qquad (2.5)$$

The weapon system performance of the Y force is dependent upon the time duration of the battle (t), the number of targets X, and the number of shooters Y such that the attrition rate coefficient $A = A(t,x,y)$. Equation (2.5) defines the most general form of the basic Lanchester equation for combat between homogeneous forces (18:13).

The modeling approach presented thus far has focused on the basic Lanchester-type model for homogeneous combat. The complex nature of a combined arms battle neccessitates the development of attrition rate coefficients for combat between heterogeneous forces. Heterogeneous combat concerns the interactions of several diverse or similar weapon systems Such systems may inflict or sustain casualties at different rates. The attrition process for heterogeneous combat is shown at figure 2. The fundamental assumptions that allow for

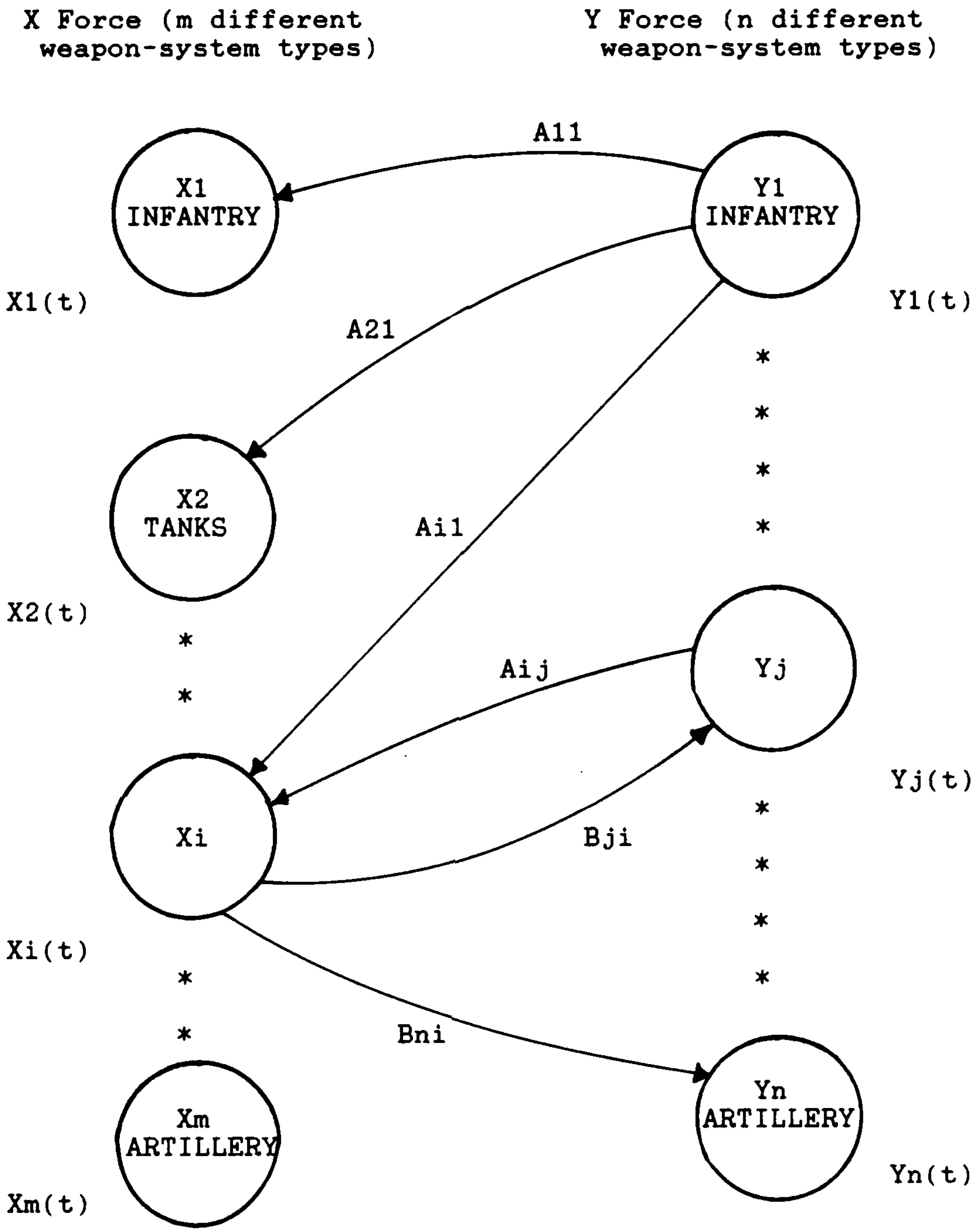

Figure 2. Heterogeneous Force Combat

Schematic showing the notation convention for indexes on the attrition rate coefficients for heterogeneous force combat. The first index denotes the target type and the second index denotes the firer type. For example, Aij is the rate at which a typical Yj firer kills Xi targets in the opposing force (18:16).

the determination of heterogeneous attrition rate coefficients are:

(A2) The attrition rate effects of various different enemy weapon-system types against a particular friendly target type are additive, and;

(A3) The loss rate of a particular friendly target type to each enemy weapon-system type is equal to the product of the single-weapon-system-type kill rate and the number of enemy firers of that particular enemy-firer type (19:55).

For heterogeneous forces, the generalization of equation (2.1) is given by the equation (2.5) (18:84). The basic equation (2.1) is expressed as:

$$\frac{dXi}{dt} = -\sum_{j=1}^{n} (Aij)*(Yj) \quad \text{with } Xi(0) = X_i^0 \qquad (2.6)$$

where

Aij = the rate at which a single Yj firer kills Xi targets

Xi = X force targets of type (i)

Yj = Y force firers of type (j)

Xi(0) = number of targets in X force of type (i) at time (t) = 0 (19:54).

The term Aij is called a heterogeneous-force Lanchester attrition rate coefficient and denotes the effectiveness of one Yj firer against Xi targets and the rate at which the attrition of the X force occurs in heterogeneous combat. Let us now define the attrition of the X force over time as the weapon system performance capability of the Y force. Using the general form of the basic Lanchester equation (2.5) the capability of the Y force can be expressed as:

$$\int_t \frac{dx}{dt} = \int_t Aij\ (t,x,y)\ dt \qquad (2.7)$$

In terms of a combat engagement between heterogeneous forces where A = Aij(t,x,y) (17:85), the performance capability of a tank unit over time can be expressed as:

$$\int_t \frac{dx}{dt} = \int_t (A * X * Y)\ dt \qquad (2.8)$$

where

A = the single-weapon-system kill rate at which the Y force attrits the X force (Lanchester attrition rate coefficient)

X = the number of targets available in the X force

Y = the number of firers available in the Y force

Let the assumption be made that the variables in equation (2.8) represent the combat engagement between heterogeneous forces of United States (Blue) and Soviet (Red) tank systems. Equation (2.8) can now be expressed as:

$$\int_t \frac{dx}{dt} = \int_t \left[(Aijk)*(Bi)*(Rj)*(Mk)\right]\ dt \qquad (2.9)$$

where

Aijk = the rate at which type (i) Blue tank attrits type (j) Red tank while performing mission (k)

Bi = the number of type (i) Blue tanks in the unit engaged at time

Rj = the percent of the Red force composed of type (j) tanks

Mk = the percent of time that the Blue unit is performing mission type (k)

Equation (2.9) defines the weapon system performance capability of a Blue tank engaged in combat with a Red tank for a time duration of t(0) to t(1). Equations (2.8) and (2.9) are equivalent as expressing the same general form of the enhanced Lanchester equation (2.5). Equation (2.9) can be adopted as the basic functional form that defines the value of a Blue tank unit engaged in combat with a Red tank unit. Equation (2.9) is the measure of capability of the Blue force that will be incorporated in the analysis of both the cumulative and unit efficiency calculations conducted in chapter III.

The practical use of equation (2.9) depends upon the ability to obtain realistic values for the variables present in the model (19:38). Data for the values of B(i), R(j), and M(k) were made available by the Armor School through the Army Tank Program Analysis study and the Army Investment Strategy study. The determination of the attrition rate coefficients will be based on the methodology of Bonder for modeling Lanchester type attrition models (20:45-56).

The value for the Lanchester attrition rate coefficient can be expressed as the reciprocal of the expected time it takes for a weapon system to kill an opposing weapon system. This value can be expressed as:

$$A_{ij} = \frac{1}{E[T_{X_iY_j}]} \qquad (2.10)$$

where

$E[\bullet]$ = mathematical expectation

$T_{X_iY_j}$ = the time for a Y firer type (j) to kill an X target type (i) (a random variable) (19:55)

The justification for this relationship is based in the hypothesis that:

> Combat is a complex random process, but it contains enough regularity that the appropriate Lanchester-type equations are a good approximation to the main course of combat (18:141).

Taylor states that the determination of the attrition rate coefficient for a hypothesized combat engagement (19:49) can be accomplished using the fitted parameter analytical model approach (7; 19:49). The fitted parameter analytical model approach assumes that valid coefficients of the attrition process in a combat engagement can be derived from the maximum liklihood estimates of attrition of forces from a Monte Carlo combat simulation. The fitted parameter modeling methodology was incorporated in both the Army Tank Program Analysis and Army Investment Strategy studies to derive the values for the loss exchange ratios. The loss exchange ratios were based on the statistical analysis of results from the CARMONETTE land combat model simulation of battalion level engagements of thirty minute durations. As noted in the preface to this thesis, the values for the attrition rate coefficients used in chapter III are strictly hypothetical and facilitate demonstration of the methodology.

Time Value of a Combat Unit

A methodology that incorporates the conceptual time value of a combat unit into the modeling process is currently being studied by the Department of Operations Research at the Naval Postgraduate School, Monterey, California (14). Under development at the Naval Postgraduate School is the AirLand Research Model. The goal of the AirLand Research Model team is to develop those methodologies appropriate for modeling Corps level combat under the AirLand Battle doctrine of the United States Army (14:1). AirLand battle doctrine dictates that the battle area can be divided into three distinct areas of operation. The three areas are commonly referred to as the rear area battle, the close-in battle, and the deep battle. The battlefield is no longer considered only the close-in area of the Forward Edge of the Battle; formerly known as the Main Battle Area. In a highly mobile environment, the battlefield can be defined by both factors of time and distance. The commander's area of concern is that area in which he can quickly and effectively move his fighting assets to engage the opposing force. Therefore, the geographic area of the battlefield can be described in terms of the time it takes for a combat unit to close with the enemy. Conceptually, a combat force engaged with an opposing force exhibits a value referred to as the force measure of effectiveness or capability. The basic measure of capability for heterogeneous armor forces was derived in the preceeding section of this chapter. Units that are not decisively engaged in the close-

in battle can be assumed to possess a discounted time value based upon the unit's ability to become engaged at some time in the future. The fundamental concepts upon which this assumption is based are presented here for the purpose of incorporation into the armor fleet measure of capability.

Several fundamental assumptions underlie the development of the time value of a combat unit (14:3).

(A4) The purpose of an army is to wage war, and therefore the only elements/units that have inherent value are fighting elements, i.e. maneuver and fire support (14:3).

(A5) The value of combat support/combat service support units derives totally from the increase or decrease in value they provide to the combat (inherent value) units they support (14:3).

Although the time value of a combat support unit is not explicitly included in the armor fleet measure of capability (equation 2.9), it is at once obvious that two conditions exist. Combat support units have no inherent value in the effort to wage war because they are not maneuver or fire support assets. The ability of a maneuver or fire support unit to effectively engage its war fighting capability over time is dependent upon the coordinated effort of support units to sustain the fight.

The following assumption forms the functional basis upon which the time value methodology is derived:

(A6) Uncommitted units and usable, but unused, support are analogous to financial assets which mature at some time in the future- that is their current value is a **discounted** version of their nominal (inherent or derived) value (14:4).

The units that are not directly engaged at the start of the close-in battle still possess a potential for use at some point in the future. This of course is dependent upon the assumption that their arrival and engagement in the close-in battle is desirable as a military course of action. Simply stated, units may have potential to influence the battle but may never realize their full combat value if they never get to the fight. Assumption (A6) is applicable to those units designated to fill the Pre-positioning Of Materiel Configured to Unit Sets (POMCUS) in the Central European battle. In addition, those units in the Continental United States (CONUS) destined for the European battle have a potential value based upon their projected ability to enter the fight at some time in the future. The capability of both POMCUS and CONUS units to influence the close-in battle can be discounted to account for their time value based upon the methodology developed in this section.

The capability measure of the armor force expressed by equation (2.9) is a measure of the basic inherent value of the fleet and the individual type tank units that comprise the total force.

> **Basic Inherent Value** is that value possessed by a maneuver or fire support unit, in contact, as a direct result of the unit's ability to conduct combat operations (14:4).

In an aggregated sense, equation (2.9) is an extension of the above definition as it applies to the armor fleet effectiveness methodology for the European-based units that become engaged at the outbreak of conventional war in Central

Europe. The value of POMCUS, and CONUS units can be defined as a situationally dependent value.

> The **Situationally Dependent Value** of a unit is its basic value, either inherent or derived, decremented by an exponential factor based on the time interval before that unit is available for commitment or can provide support (14:6).

The situationally dependent value of a unit can be expressed in mathematical terms based on the following contention. The armor capability measure of the engaged combat unit defines the unit's basic inherent value at the start of the engagement, i.e. at time $t(0)$. Let us assume that the basic value (either inherent or derived) of a unit at time $t(0)$ is given by the expression (14:6):

$$V = V\ (s(\ t(0)\ )) \qquad (2.11)$$

where

V = value of the unit, inherent or derived

s = state of the unit at time (t)

If the unit in question were not available for combat until some time in the future where $t > t(0)$, then the future inherent value of the unit can be discounted back to the present and can be expressed by the formula:

$$V = V\ (s(t(0))\ e^{-C\ (\ t\ -\ t(0)\ )} \qquad (2.12)$$

where

$$e^{-C\ (\ t\ -\ t(0)\ )} \qquad (2.13)$$

defines the discount factor and C is the decay constant that

is used to determine the present value of a combat unit's future inherent value. The solution of (2.13) is straightforward. In order to determine the present value of a combat unit's inherent value, an important assumption must be made. Assume that combat units that are within 90 days of entering the close-in battle have a negligible value of 0.05. This value is a completely arbitrary assignment. The value of the decay constant C is determined by solving:

$$\exp(-90 * C) = 0.05$$

which reduces to:

$$C = \frac{\ln 0.05}{-90} = 0.0333$$

Assume that a POMCUS unit can be constituted and become operational in 30 days. Then the present value factor of the future inherent value of the POMCUS unit is equal to:

$$Z = \exp(-0.0333 * 30) = 0.368 \qquad (2.14)$$

Similarly, assume a CONUS unit can enter the close-in battle in 120 days. The CONUS unit present value factor at the start of the battle is equal to:

$$Z = \exp(-0.0333 * 120) = 0.018 \qquad (2.15)$$

The Z parameters determined in (2.14) and (2.15) are present value coefficients for POMCUS and CONUS units, respectively. The parameters represent a weighting of the present value of the unit's future combat capability.

The value expressed by equation (2.11) is the instantaneous value of a unit at time (t) (14:6). When t = 0 equation (2.12) becomes, by definition of inherent value, equation (2.9). However, the state of engaged combat units changes with time mainly due to attrition. Therefore, to maintain consistency with the concept of exponential future discounting, the following equation expresses the unit's measure of average value (14:7):

$$G(V) = Z \int_{0}^{\infty} V\,(s(\ t(0)\ )\ e^{-Ct}\ dt \qquad (2.16)$$

where the current time is (t) = 0. It is now apparent that at the start of the engagement, i.e. where (t) = t(0) = 0, then equation (2.16) reduces to equation (2.11) for committed units and Z = 1. The value of Z for all other units not in contact is determined as before and the present value of these yet to be committed units can be determined by combining Z with equation (2.9) to yield the expression:

$$\text{CAPABILITY} = \sum_{i}\sum_{j}\sum_{K} Z * \int_{t} \Big[(Aijk)*(Bi)*(Rj)*(Mk)\Big]dt \qquad (2.17)$$

Equation (2.17) defines the measure of capability of any Blue unit engaged or otherwise potentially engaged with a heterogeneous Red force. Equation (2.17) can now be adopted as the enhanced Lanchester-model variable relationship that defines the armor fleet capability measure from which the efficiency of the fleet will be determined.

Efficiency Evaluation

An efficient fighting force could be characterized as that force that maximizes the use of all available resources of men and materiel to produce the largest degree of war fighting capability. In a practical sense, the commander of an armor unit must strive to use his tank assets in the most efficient way to destroy the opposing force in combat. Thus, the objective of a fighting force should be the destruction of the maximum number of enemy weapons with the tank assets available. The method of approach to the analysis of the efficient use of available tank assets must address two issues.

First, a determination must be made as to the theoretical maximum destructive capability that an armor unit could attain given some baseline parameters. For our problem, the measure of an armor unit's war fighting capability is given by equation (2.17). As the result of the interaction of United States and Soviet forces in heterogeneous combat, one should expect the number of Red tanks killed as described by equation (2.17). The measure of capability then becomes the theoretical output for that Blue armor force being evaluated.

The second issue to analyze is how well the unit can use its available tank assets, relative to all the units participating in the battle engagement. Relative capabilities are determined by solving equation (2.17) for each armor unit. An average fleet capability measure can be determined

and then the distribution of assets can begin in an effort to increase the fleet capability average. The capability measures for the armor units evaluated in this thesis are shown at Appendix B.

The relative efficiencies of the units evaluated in this thesis, based upon an analysis of the output capability of the units and the input of tank assets available to the unit can provide insights into the deployment of those assets. As the methodology of efficiency analysis is developed here, the objective of the analysis must remain clearly defined. The objective of the efficient use of armor assets involves the management and conservation of these assets to achieve the largest war fighting capability. The method of analysis presented here offers to quantitatively determine those units that are efficient relative to the entire force evaluated. Determining those units that are not efficient relative to the entire fleet demonstrates where attention must be given to the proper management and deployment of tank assets.

The efficiency of a production process has traditionally been measured by the ratio of the amount of output from a production process to the amount of input to the production process. The identification of the inputs and outputs to the production of the war fighting process is relatively easy. Description of the war fighting "production" process or function itself is more difficult. In a broad sense, the production function can be defined as the interaction of hetereogenous combat forces, each with the intention of

attaining the goal of defeating the opposing force. The process is carried out by the strategic movement of forces and the tactical engagement of men and materiel in an effort to "win" the war. The measure of how well a combat unit carries out the tactical engagement is difficult to ascertain from the analysis of individual inputs to the process. Imbedded in the process are such factors as unit morale and esprit, unit training readiness, maintenance of individual and crew equipment, and command leadership which all interact toward "winning" the combat engagement. It must be assumed that all units exhibit equal measures of the above inputs, i.e. all armor units are equally trained, maintained, and commanded by capable leaders. (16)

Another measure of input that is embedded in the war fighting process is the assessment of the attrition of forces. Because there exists no historical data base from which explicit weapon on weapon atttrition rates can be derived, the use of combat modeling is necessary to determine the attrition rate coefficients. It must be understood that no combat simulations can ever duplicate the realities of a combat engagement. Therefore, the practical use of combat modeling rests with the intentions of what the model is supposed to simulate. For this thesis, the analysis focuses on the attrition of theater-level forces engaged in hetereogenous combat. Therefore, the attrition rates used in this analysis should reflect the interaction of theater-level forces.

The discussion to this point forms the basis for the analysis of the relative efficiency of armor units. The traditional ratio of relating output to input in a production process serves as the starting point. For the armor force efficiency problem, the output for any armor unit is the theoretical measure of capability described by equation (2.17). The measure of capability described by equation (2.17) reduces to the number of Red tank kills that the Blue unit is expected to destroy in combat. For a given Blue combat unit of type (i) tank, the capability of that unit relative to each type (j) Red tank, can be determined by equation (2.17).

The input to the efficiency ratio is found by analyzing the inputs to the capability measure of equation (2.17). The attrition rate coefficient (Aijk) is a variable parameter that defines the combat interaction of the Blue and Red forces. The Red fleet variable (Rj) is an expected value derived from classified studies. Since the (Aijk) and (Rj) are uncontrollable they will be regarded as fixed in this thesis. The Blue tank (Bi) is an input variable in the sense that this resource is semi-fixed. It is a constant number of tank assets in an armor unit. However, using (Bi) as the input parameter to the efficiency evaluation proposes to offer insights as to how this asset can best be managed. The management of these assets involves the sensitivity analysis of how and when these tanks should become engaged in combat. The mission profile of the unit (Mk) and the time

value coefficient of the unit (Z) can be analyzed for sensitivity to the overall capability measure in terms of the efficiency in the use of the unit's tank assets.

The efficiency relationship expressed as the ratio of the output to the input is written as:

$$\text{EFFICIENCY} = \frac{\text{OUTPUT}}{\text{INPUT}} = \frac{\text{UNIT CAPABILITY}}{\text{NUMBER OF TANKS IN UNIT}} \tag{2.18}$$

(2.18) describes the efficiency measure as the ratio of the expected capability for an armor unit divided by the number of tanks available in the unit. The measure relates how well the unit can use available assets to achieve the maximum possible output. The methodology used to quantitatively analyze equation (2.18) is found in understanding the theory and application of data envelopment analysis (DEA).

The model that serves as the basis for data envelopment analysis was developed by Charnes, Cooper and Rhodes (8:429). The Charnes, Cooper, and Rhodes (CCR) data envelopment model is based on the efficiency models and extensions to the work of Farrell (10). The so called Farrell efficiency model will serve as the basis for presenting the theory of DEA.

Farrell studied the production efficiencies of several firms by analyzing the observed value of inputs and outputs in the production process. A simple example of Farrell's approach is presented here and shown in figure 3. Figure 3 illustrates a production process that is characterized by a single output and two input factors to production. The curve

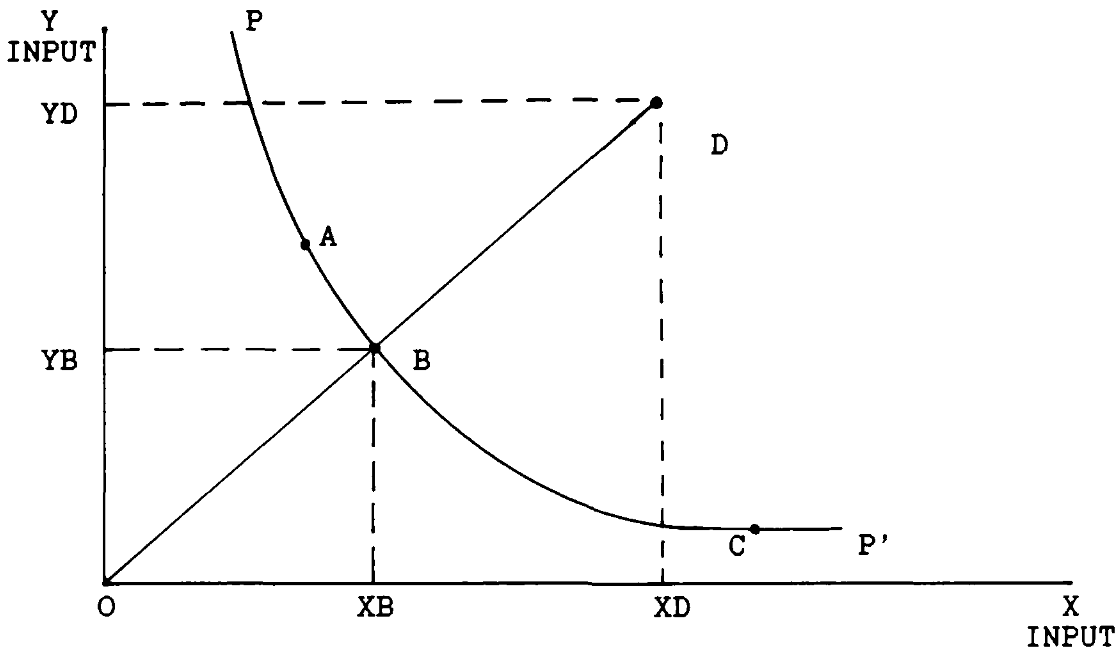

Figure 3. Farrell's Technical Efficiency

of PP' represents a production frontier where various quantities of the inputs of X and Y are used by efficient producers to yield one unit of output. It must be assumed that producers will always produce the highest level of output with any set of input combinations. Therefore, the curve PP' represents a production frontier of perfectly efficient producers and no points between the origin and PP' are attainable, if the assumption stated above is maintained. The point at D represents a producer who produces one unit of output with input combination XD and YD. A line drawn fron the orgin to point D now represents the relative efficiency of producer D to the other producers on the production frontier.

Producers B and D use the same proportionate mix of inputs, however producer B is efficient and producer D is not. In determining the efficiency of D to B then, the ratio of the distance OB to OD represents the "technical efficiency" (10:254) of producer D. Thus, the efficient producer B uses (XB,YB) inputs to produce one unit of output. Producer D uses (XD,YD) inputs but produces only OB/OD times as much as producer B. Thus, with its current level of input, producer D would be expected to produce OD/OB times as much as its current output if it were an efficient producer. Likewise, producer D, if efficient, could produce one unit of output with a fraction of its current inputs (XD,YD) as shown by the relative efficiency of D to B. The basis for the efficiency assessment of a producer is now revealed. In an inefficient organization, either the same output can be produced with reduced input conbinations or greater output can be produced with the same input combinations used in an efficient manner. For military budget analysts and planners, the prospect of getting more out of the current level of input resources is the motivation for including this efficiency methodology in the armor force problem. The methodology presented here proposes to demonstrate where possible trade-offs between inputs can result in a more efficient and effective armor force.

Farrell contends that the determination of the production frontier PP' is a difficult or impossible task when there exists little or no information concerning the

production function (9:46). Farrell used the observed level of production of a number of firms to derive a production frontier, as illustrated in figure 4.

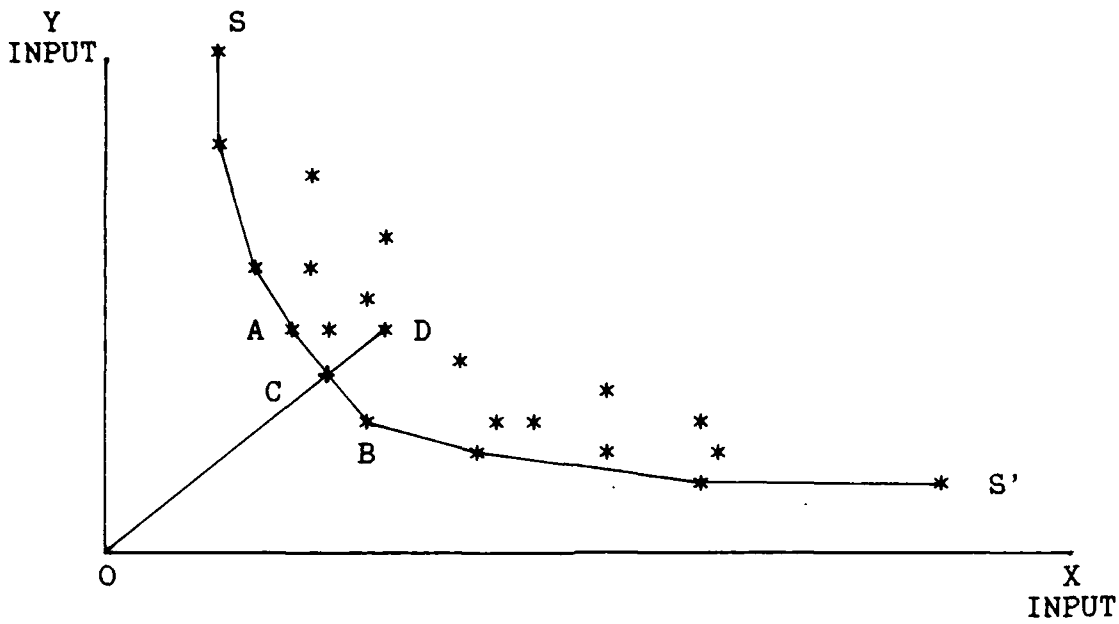

Figure 4. Farrell's Production Frontier

Various producers using various input combinations of X and Y will result in a scatter plot of outputs of the production process. The production frontier SS' is constructed as a piece-wise linear isoquant. Points on the scatter plot are connected such that the slope of the line is nowhere positive and there exist no points between the piece-wise linear frontier and the origin. The curve SS' then represents the estimate of the efficient isoquant for this production process (10:256). The relative efficiency of producer D can

be determined by comparing it to the hypothetical producer at point C, whose location represents a linear conbination of points A and B on the efficiency frontier. The technical efficiency of D is represented by the measure of the ratio of the distance OC to OD which is less than unity. Simultaneously, the efficiency of D is relative to the efficient producers A and B. Thus, the producer's shortest ray distance from the efficiency frontier represents a relative efficiency measure (10:255-256).

The approach can be expanded to include production frontiers characterized by multiple inputs and multiple outputs. In this case, the idea of a production isoquant is represented by a plane of points in N-space, and frontiers are no longer edges but now facets on a hyperplane. Each producer is represented by a point in N-dimensional space as the result of production process of N input variables. The facet is that part of the hyperplane "whose points can be expressed as weighted averages, with nonnegative weights, of N defining points" (10:257). The relative efficiency of a producer is now represented by the distance from the measured output of the producer to the facet of the efficiency surface in N-space.

Charnes, Cooper, and Rhodes devised the DEA model in the effort to formulate and solve complex, nonlinear relationships characteristic of the ratios that define the efficiency measure (8). Similar to the method used by Farrell, DEA uses all producing units simultaneously to

determine the relative efficiencies of each individual producer. The definition used by CCR to describe the DEA model states:

> (D1) Our proposed measure of the efficiency of any DMU (decision making unit) is obtained as the maximum of a ratio of weighted outputs to weighted inputs subject to the condition that the similar ratios for every DMU be less than or equal to unity (8:430).

DEA theory makes no prior assumption on the nature of the production process nor the values for the weights of the inputs and outputs to the process. In the notation of CCR the mathematical programming problem is formulated as follows:

$$\text{Max (hk)} = \frac{\sum_{r=1}^{s} (Ur)*(Yrk)}{\sum_{i=1}^{m} (Vi)*(Xik)} \qquad (2.19)$$

Subject to:

$$\frac{\sum_{r=1}^{s} (Ur)*(Yrj)}{\sum_{i=1}^{m} (Vi)*(Xij)} \quad <= 1$$

$$(Ur) \text{ and } (Vi) \quad >= 0$$

where

j = 1, ...,n, (n total DMU's)

r = 1, ...,s, (s total outputs)

i = 1, ...,m. (m total inputs)

(8:430)

The variables (Yrj) and (Xij) are the known outputs and inputs of the (jth) DMU and (Ur) and (Vi) >= 0 are the variable weights solved for when all DMU's are considered in the data set. The variables (Yrk) and (Xik) are the outputs and inputs of the DMU being evaluated for its efficiency rating relative to all (j) DMU's (8:430).

Equation (2.19) represents maximizing the function of the ratio of weighted outputs to weighted inputs, consistent with the definition (D1).

CCR state that "the above formulation is an extended nonlinear programming formulation of an ordinary fractional programming problem" (8:431). The model in (2.19) may be replaced by a linear programming problem by means of the theory of linear fractional programming (6:1358). Equation (2.19) can thus be written, in equivalent form, as:

$$\text{Min (gk)} = \sum_{i=1}^{m} (Vi)*(Xik) \tag{2.20}$$

Subject to:

$$0 <= \sum_{i=1}^{m} (Vi)*(Xij) - \sum_{r=1}^{s} (Ur)*(Yrj)$$

$$1 = \sum_{r=1}^{s} (Ur)*(Yrk)$$

$$\varepsilon <= \quad (Vi) \text{ and } (Ur)$$

where ε is a very small (infinitesimal) quantity (9:19) to

insure (V_i) and (U_r) are positive and (g_k^*) = minimum (g_k), the reciprocal of the efficiency index defined by $(h_k^*) = 1 / (g_k^*)$ (6:1358). The reader is invited to see Charnes, Cooper, and Rhodes (8) for a more detailed explanation of the development of the linear fractional equivalent form.

For the purpose of this thesis, Charnes' "decision making unit" or DMU is synonomous with the unit or major field command that comprise the data set contained in Appendix B. The relative efficiencies of these major field units will be determined based on the DEA methodology of Bessent using the model of equation (2.20).

To determine the relative efficiency of a unit, it is included in the model in the objective function as well as in the constraint equations to insure that an optimal (h_k^*) = maximum (h_k) will always satisfy $0 \le (h_k) \le 1$ (4:1079). Each unit is evaluated relative to all other units considered and the $j = 1, \ldots n$ constraints insure no unit has an efficiency rating greater than unity. For a perfectly efficient unit then, $(h_k^*) = 1 =$ maximum (h_k) (6:1358).

According to Bessent and the model of equation (2.20), a value for (g_k^*) greater than one represents an inefficient unit since $1 / (g_k^*) = (h_k^*)$ will always be less than one. If a value of (g_k^*) greater than one or the presence of positive slack (S_i-) or surplus (S_r+) variables is found in the non-basis of the solution, then the conditions indicate a source of inefficiency (6:1359). Efficiency can be attained

if the results above are applied to the initial input and output values in the form:

$$
\begin{aligned}
(\hat{X}ik) &= (Xik) - (Si-) && i = 1, \ldots m \\
(\hat{Y}rk) &= (Yrk)*(gk*) + (Sr+) && r = 1, \ldots s
\end{aligned}
\qquad (2.21)
$$

where

$(\hat{X}ik)$ = input "value if efficient"

$(\hat{Y}rk)$ = output "value if efficient"

$(gk*)$ = minimum (gk) from model (2.20)

$(Si-)$ or $(Sr+) > 0$ represent the shadow prices of the non-basis variables associated with the (i) and (r) input and output, respectively (6:1359).

Simply stated, if the initial input and output values are modified consistent with equation (2.21), then the new values of (Xik) and (Yrk) would make the unit evaluated efficient. Particularly important for this thesis will be the determination of (Yrk), the output value if efficient. From this measure the analysis of input trade-offs will allow the analyst the ability to determine where increased output from available resources can be accomplished.

Examination of the variables in equation (2.17) is necessary to identify which parameters are candidates for resource adjustment. The nature of equation (2.17) indicates that some of the terms are infeasible for adjustment by resource planners. The attrition rate coefficients (Aijk) are fixed as the result of combat modeling and represent the heart of the production process, i.e. the interaction of heterogenous force combat. The value of the percent of Red

fleet (Rj) expected to oppose Blue forces is not open to modification. The number of Blue tanks (Bi) is a resource that can be adjusted, however, as Clark states:

> It is unreasonable and unlikely that ... commanders would be willing to reduce the input amounts as suggested and in so doing give up the extra capability and strength these valued inputs might provide in combat. (9:147)

For the purpose of this thesis, it is practically unthinkable that a unit would be willing to give up its tank assets in the interest of efficiency alone. It must be the objective then to evaluate relative unit efficiencies by identifying how existing resources can be better used in combat. The candidate variables from equation (2.17) that will be examined for sensitivity to the unit efficiency measure are the discounted time value parameter (Z) and the unit mission profile parameter (Mk).

III. Findings

Scenario and Assumptions

The findings presented in Appendix B result from the application of the unit capability measure (equation 2.17) and the data envelopment analysis model described by the equation of (2.20). The discussion and analysis of these findings is based upon the following assumptions and scenario conditions.

The area of operations for this thesis consists of the Central European theater. The methodology presented is not limited by the focus on Europe. As long as the attrition parameters are consistent with the theater of operations and the battlefield environment the methodology could be applied to any area of operations.

It is assumed that the duration of the battle engagement is at least 180 days. This assumption is critical when considering the length of time necessary for the air and sealift assets to deliver POMCUS and CONUS units to the theater. For our purposes, the capability measures shown in Appendix B are the expected daily attrition of Red tanks by the United States units listed in numerical order.

It is assumed that a POMCUS unit can be delivered and become operational within 21 days of the start of the battle engagement. It is assumed that a CONUS unit can be delivered and become operational within 90 days. CONUS units include National Guard and Army Reserve units destined for the European theater. The discounted present value coefficient

for a POMCUS unit is $Z = 0.4966$. The discounted present value coefficient for a CONUS unit is $Z = 0.0498$.

The attrition parameters are based on the Systems Effectiveness Model inputs to the CARMONETTE model used in the Army Investment Strategy study. These "attrition" parameters will subsequently be updated once the theater-level modeling results are obtained from the Army Concepts Analysis Agency. The parameters used in this thesis are hypothetical and facilitate the demonstration of the methodology and its application to force planning. The attrition rate coefficients are based on a defensive mission profile and are summarized in table I below.

TABLE I

Attrition Rate Parameters

Blue 1 against Red 1 = 1.30	Blue 3 against Red 1 = 2.11
Blue 1 against Red 2 = 1.20	Blue 3 against Red 2 = 2.01
Blue 1 against Red 3 = 0.74	Blue 3 against Red 3 = 1.48
Blue 2 against Red 1 = 1.75	Blue 4 against Red 1 = 2.64
Blue 2 against Red 2 = 1.63	Blue 4 against Red 2 = 2.51
Blue 2 against Red 3 = 1.00	Blue 4 against Red 3 = 1.85

The attrition rate coefficients reflect the expected number of Red tank (j) kills per Blue tank (i) per day.

The Red fleet profile is shown at Appendix A. Three types of Soviet tanks were modeled and the yearly fleet composition is the expected percentage consistent with the threat estimates contained in the Army Investment Strategy study. The figures shown were used for the input variable

R(j) in the unit capablity measure of equation (2.17).

The Blue fleet profile for B(i) is contained in Appendix B. For each unit identified in the far left column (1-25) the number of Blue 1, Blue 2, Blue 3, and Blue 4 type tanks on hand are listed under the appropriate heading. The numbers shown are consistent with the sample fleet distribution plan in the Army Investment Strategy study. The only exclusions were those units designated as training base, Korea, or Army Reserve and National Guard round-out units destined for non-NATO theaters. The units designated one through twenty five represent European based, POMCUS, and CONUS units available for action in the Central European theater.

It is assumed that the personnel of those units that fill the POMCUS stocks initially can be replaced in time to facilitate the movement of a cohesive and complete CONUS unit. The implication here is that the wartime program of instruction for the training of armor crewman must allow for the soldier to become a fully trained and operational member of a CONUS "stocked" unit well prior to the 90 day delivery target assumed in this initial scenario.

Findings

The unit capability measure expressed by equation 2.17 was used to determine the expected number of Red tank kills per day for each of the 25 units listed in Appendix B. Calculations were performed on the sample distribution option for the years 1987 to 2000.

The capability of the units based in Europe represents the basic inherent value of the unit at the start of the engagement. The situationally dependent values for the capability of POMCUS and CONUS units represents the results of exponential decay due to the time lag necessary for the delivery of these units to the engagement area of operations. The measure of capability for European based units is much greater, relative to the input number of tanks, than the capability measure of either a POMCUS or CONUS unit. Since equation (2.17) is a deterministic model, the results are consistent with our expectations as long as Central Europe is the area of interest. Based upon capability alone, it is no surprise that the results support the forward fielding of armor units into the area of expected operations.

The relative unit efficiencies shown in Appendix B were determined using the LPV2 micro-computer linear programming software package (16). An example linear programming formulation for the determination of the relative efficiency of unit 14 for year 1987 is shown at Appendix C. The output summary for unit 14 is shown at Appendix D. Each of the 25 units for the years 1987 to 2000 was evaluated by applying the data envelopment model of equation (2.20). It is clearly evident from the unit efficiencies that European based units are nearly perfectly efficient in all cases in their employment of tank assets. Again, in that equation 2.17 is deterministic and represents a theoretical production function for the tank killing capability of United States

units, the case for forward fielding is supported by the unit efficiency evaluations. Units based in the theater of interest form the production frontier for all the units considered in this thesis. Examination of the relative unit efficiencies reveals that the efficiency of the unit is directly related to the time value associated with that unit's situationally dependent value. The present value coefficient (Z) for European based units is equal to one. The relative unit efficiencies for European based units are nearly one and in some cases are perfectly efficient. The present value coefficient (Z) for POMCUS units is equal to 0.4966. The relative unit efficiencies for POMCUS units closely approximates this value throughout the time period 1987 to 2000. The average unit efficiency for POMCUS units is equal to 0.5064. The relative unit efficiencies for CONUS based units closely approximate the present value coefficient for those units ($Z = 0.0498$). It is evident that the unit efficiency evaluations are sensitive to time. The (Z) parameter thus becomes the focus of sensitivity analysis later in this chapter.

According to the method of Bessent (equation 2.21) the results from the DEA model can be used to determine how an inefficient unit's assets can be modified or reassigned to move that unit toward the efficiency frontier. Examination of the relative unit efficiency for unit 14 (Appendix D) will serve as an example for the determination of the capability of unit 14 if it were a perfectly efficient unit.

The relative unit efficiency of unit 14 was determined to be 0.049990. This figure is the reciprocal of the minimum value of the objective function and was solved for consistent with the method of Bessent. Efficiency can be attained for unit 14 if we apply the results in Appendix D to the original data by solving for the "values if efficient" of the input-output combination. The range of cost coefficients for non-basic variables and the solution to the dual linear programming problem indicate that all the slack variables (Si-) are less than zero. The minimum value of the objective function (gk*) is greater than one in that (gk*) = 20.00368 indicating inefficiency in unit 14. If these results are applied to the original data in the form of equations (2.21) then the "values if efficient" for unit 14 can be determined by solving:

$$(Xik) - (Si-) = (Xik)$$

or;

$$252 - 0 = 252 \qquad \text{(efficient input)}$$

and simultaneously;

$$(Yrk) * (gk*) + (Sr+) = (Yrk)$$

or;

$$10.88 * 20.00368 + 0 = 217.64 \qquad \text{(efficient output)}$$

In other words, the unit 14 input combination of 252 Blue 2 tanks is efficient however the unit is not producing the highest output possible represented by the value if efficient

of 217.64 Red tank kills.

The information revealed for unit 14 now becomes important to the analysis of the force structure and the employment and deployment plans for the unit as well as the entire armor fleet. Although modifications to the input of 252 tanks for unit 14 were not necessary in this instance, such decisions must be made consistent with current or projected production and distribution guidelines. The important result indicated for unit 14 is the fact that the unit is not using the 252 tanks in the most efficient manner, as evidenced by the 0.049990 efficiency measure. Using the DEA methodology as the basis for analysis, the decision maker and analyst can examine how variations in the time value of the unit and the unit's assigned mission profile influence the overall efficiency in the output production of Red tank kills.

Sensitivity to Time Value and Mission Profile

By modifying the hypothetical delivery times for both POMCUS and CONUS units, the analyst can assess the results on both unit capability and relative unit efficiency. Appendix E shows the results of such a modification. The unit capability and efficiency results reflect the change in the time necessary to deliver a POMCUS unit from 21 to 14 days and the time to deliver a CONUS unit from 90 to 60 days. The relative unit efficiencies are still nearly perfectly efficient for the European based units and these units form the efficient production frontier. POMCUS units have increased their unit

efficiencies by 13 percent and unit 14 has increased its unit efficiency by 8.5 percent from 0.049990 to 0.135637, all relative to the efficient frontier. The unit efficiencies still reflect the present value coefficients of both POMCUS and CONUS units as do the European based units. The analyst can determine very easily the impact of how a change in the expected delivery time of non-theater units will influence the efficiency determination of employed tank assets. The relative unit efficiencies closely approximate the Z parameter that is input to the model.

It has been demonstrated that the methodology outlined in this thesis can allow the analyst and the decision maker the ability to interact in near real time to answer both distribution and deployment questions. Data envelopment analysis can assess where input assets must be modified to achieve unit efficiency. The methodology also allows for the assessment of how changes in the delivery times affect both capability and efficiency. The impact of changes in the employment of tank assets can also be assessed by using the data envelopment methodology.

Appendix F shows the relative unit efficiencies that result when both delivery times and employment parameters are modified in the model of equation (2.17). For this modification, POMCUS delivery times are changed from 21 to 14 days, CONUS delivery times are changed from 90 to 60 days, and the mission profile of percent time in the defense for all non-European based units changed from 0.50 to 75 percent time

in the defense. The results indicate a 31 percent increase in the unit efficiencies of POMCUS units and nearly a 7 percent increase in the efficiencies of CONUS units, all relative to the efficiency frontier. Unit number 14 is now 20.35 percent efficient relative to the entire fleet. The average fleet efficiency has increased over 12 percent from the initial 1987 results illustrated in Appendix B. The results shown in Appendix F indicate that a higher degree of overall fleet efficiency can be achieved if the non-European based units are employed more in the defense (75 percent of the time) relative to the European based mission profile of 50 percent time in the defense.

It has now been demonstrated that the relative unit efficiencies are sensitive to delivery deployment time and mission employment decisions. There exist an infinite number of input combinations that could be assessed using the model of equation (2.17) and the data envelopment methodology outlined in this thesis. The utility of the analytical framework presented in this study is clearly evident. The analyst can produce the results of critical distribution, deployment, and employment modifications in a timely and accurate manner. As has been revealed by the findings, the use of data envelopment analysis allows the analyst a research tool with which to compare different tank distribution and production options in terms of achieving the most efficient deployment and employment of armor assets.

IV. Summary, Recommendations, and Conclusions

Summary

The analytical framework for the determination of the preferred main battle tank fleet developed in this thesis has incorporated several mathematical models into a useful research method. The use of the Lanchester equations of heterogenous combat serves as the basis of the model described by equation (2.17). The use of attrition rate coefficients based on theater level modeling proposes to approximate more closely the expected attrition of Soviet tanks by United States forces. The use of attrition rate coefficients incorporates the time sensitive nature of the attrition process. Coupled with the situationally dependent time value of a combat unit, the model of equation (2.17) fully and more accurately defines a unit's combat capability. It has been shown that a unit not engaged in combat has only negligible value to the entire fleet. As long as a unit is not engaged, it is not employing its combat assets in an efficient manner. The findings support the contention that only units engaged in the theater of operations are operating efficiently and producing the highest output possible. The forward fielding of units in the area of the highest potential threat is supported by the near perfect unit efficiencies of the European based units considered in this thesis. It is also evident that the pre-positioning of combat assets near an expected area of operations increases a unit's ability to engage its combat power more efficiently. The

analytical framework using the data envelopment methodology allows the analyst the ability to conduct trade-off analysis of the input parameters to determine the most preferred distribution, deployment, and employment option. The method presented in this thesis allows for answering the questions of where combat assets should be fielded, in what quantity, and how their deployment and employment reflect the efficient use of those assets. Finally, the methodology gives the analyst a research tool with which he can quickly and accurately determine the impact of various input combinations upon fleet capability and fleet efficiency. The methodology is adaptable for use on a micro-computer. The unit capability results illustrated in Appendix B were obtained by executing a simple PASCAL computer program. The unit efficiencies were determined by executing the Sunset Software LPMIP83 Linear Programming System on a micro-based computer. In summary, all of the subsidiary objectives outlined in chapter I of this thesis have been successfully achieved.

Recommendations and Conclusions

It is the recommendation of this author that the United States Army Armor School adopt the analytical framework and methodology presented in this thesis research. The comparison of several varied and diverse distribution and production plans can be easily accomplished based upon the most efficient use of current and projected armor assets. At the same time, the sensitivity of the situationally dependent time value and the employment mission profile for individual

units allows force planners the ability to decide how fast and in what manner individual units must be employed to attain the highest degree of the efficient production of combat capability.

The methodology presented here is not confined to the question of the efficient use of tank assets. The data envelopment modeling approach allows the decision maker the ability to assess the efficient use of resource assets. As such, data envelopment analysis used as a research tool is applicable to many situations where clearly defined performance measures are not readily available. As long as the outputs and inputs to some production process are quantifiable, the nature of the production process itself need not be defined. Data envelopment analysis allows for the comparison of relative unit efficiencies and offers insights into how input assets can best be used to produce the highest output. The many and varied situations where Army planners and commanders could use the analytical framework presented in this thesis is left to the imagination of the operations research analyst.

The analytical framework for efficiency evaluation and determination of the preferred main battle tank fleet developed in this thesis follows the guidelines for analytic modeling established by the Military Operations Research Society. The model of equation (2.17) is a "generation" model (13:3). The product of this model is a combat measure of effectiveness representing the dynamics of combat over

time (13:3). The representative "process" model in this thesis is the methodology of the efficiency determination for each unit. The incorporation of equation (2.17) into the analytical framework for efficiency assessment represents the utilization of dynamically generated combat measures, each weighted by a present value coefficient. The "process" is then executed to assist in the answering of specific questions regarding tank production, distribution, and employment. In this way, the results generated by the solution of equation (2.17) are done so within a clear context and for a specific purpose (13:4).

The analytical framework for efficiency evaluation and the determination of the preferred main battle tank fleet is a quantitative method that can assist defense decision makers in determining where armor assets can best be deployed and in what quantities. The organization of the combat forces of the Army based on the most efficient allocation and employment cf combat resources must be the goal of Army decision makers. The combined efforts of operations research analysts, planners, and commanders to attain this goal will only serve to make the Army a more effective fighting force capable of winning the next war. The analytical methods outlined in this thesis research propose to assist the senior Army leadership in attaining the goal of arming, equipping, and training an "Army of Excellence," the most efficient and effective fighting force in the modern world.

Red Fleet Composition
(percent of fleet by year)

Weapon System	Fiscal Year	1987	1988	1989	1990	1991	1992
RED 1		81	80	79	79	76	72
RED 2		19	20	21	21	24	24
RED 3		0	0	0	0	0	4

Weapon System	Fiscal Year	1993	1994	1995	1996	1997	1998
RED 1		71	61	45	30	12	6
RED 2		24	24	24	24	26	24
RED 3		5	15	31	46	62	70

Weapon System	Fiscal Year	1999	2000
RED 1		6	6
RED 2		22	18
RED 3		72	76

Blue Fleet Capability and Efficiency

YEAR: 1987

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1		315			272.03	.999926
2		141			121.77	.999961
3		63			54.41	.999931
4		141			121.77	.999961
5		378			326.44	.999939
6	63				40.35	.999950
7	378				242.11	.999992
8	315				201.76	.999983
9	27				17.29	.999785
10	819	960			672.75	.496978
11			252		13.17	.060512
12		94			4.06	.050010
13		63			2.72	.049990
14		252			10.88	.049990
15	315				10.09	.050009
16		189			8.16	.049990
17	126				4.04	.050059
18	63				2.02	.050059
19		252			10.88	.049990
20	189				6.05	.049977
21	110	189			11.68	.049982
22	1396				44.71	.050003
23	63				2.02	.050059
24	124				3.97	.049985
25	3035	381			1129.66	.496992

TOTAL FLEET CAPABILITY.............. 3334.79

AVERAGE FLEET CAPABILITY............ 133.39

AVERAGE FLEET EFFICIENCY.................... .428161

Blue Fleet Capability and Efficiency

YEAR: 1988

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1		252	63		283.31	.999969
2		141			121.68	.999957
3		63			54.37	.999969
4		141			121.68	.999957
5		378			326.21	.999969
6	63				40.32	1.000000
7	378				241.92	1.000000
8	315				201.60	1.000000
9	27				17.28	1.000000
10	504	960	315		735.67	.523327
11			252		13.17	.050032
12			94		4.91	.050041
13	63	63			4.73	.049952
14		252			10.87	.049982
15	315				10.08	.050000
16		189			8.16	.050028
17	126				4.03	.049975
18		63			2.72	.050027
19		252			10.87	.049982
20	189				6.05	.050017
21		189	110		13.90	.064423
22	1371				43.87	.049998
23	63				2.02	.050099
24	149				4.77	.050021
25	3350	444	68		1291.32	.496999

TOTAL FLEET CAPABILITY.............. 3575.51

AVERAGE FLEET CAPABILITY............ 143.02

AVERAGE FLEET EFFICIENCY................... .429389

Blue Fleet Capability and Efficiency

YEAR: 1989

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1			315		329.02	.999962
2			141		147.27	.999962
3		63			54.33	.999965
4		141			121.60	.999954
5		378			325.99	.999971
6		63			54.33	.999965
7	63	315			311.94	.999926
8	315				201.44	.999785
9	27				17.27	.999785
10	363	960	456		763.56	.496981
11			252		13.16	.049997
12			94		4.91	.050008
13	63	63	63		8.02	.049990
14		252			10.87	.050017
15	315				10.07	.049979
16		189			8.15	.050001
17	126				4.03	.050004
18		63			2.72	.050063
19		252			10.87	.050017
20	189				6.04	.049963
21		189	110		13.89	.049983
22	1371				43.84	.049992
23			63		3.29	.049997
24	149				4.76	.049945
25	3869	444	155		1500.46	.496994

TOTAL FLEET CAPABILITY.............. 3971.83

AVERAGE FLEET CAPABILITY............ 158.87

AVERAGE FLEET EFFICIENCY................... .427728

Blue Fleet Capability and Efficiency

YEAR: 1990

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1			315		329.02	.999962
2			141		147.27	.999962
3			63		65.80	.999939
4			141		147.27	.999962
5		378			325.99	.999969
6		63			54.33	.999969
7		378			325.99	.999969
8	189	126			229.53	.999898
9	27				17.27	.999785
10	300	960	519		776.24	.496985
11			252		13.16	.049997
12			94		1.77	.018027
13	63	63	63		8.02	.049990
14		63	189		12.59	.050011
15	126	189			12.18	.050002
16		189			8.15	.050002
17	126				4.03	.050004
18		63			2.72	.050063
19		252			10.87	.050017
20	189				6.04	.049963
21		189	110		13.89	.049984
22	1371				43.84	.049992
23			63		3.29	.049997
24	149				4.76	.049945
25	4121	459	150		1584.38	.496915

TOTAL FLEET CAPABILITY.............. 4148.40

AVERAGE FLEET CAPABILITY............ 165.94

AVERAGE FLEET EFFICIENCY.................... .426452

Blue Fleet Capability and Efficiency

YEAR: 1991

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1			315		328.54	.999970
2			141		147.06	.999964
3			63		65.71	.999975
4			141		147.06	.999964
5			378		394.25	.999975
6			63		65.71	.999975
7		378			325.31	.999846
8		315			271.09	.999840
9		27			23.24	.999846
10	174	960	645		800.13	.520426
11			252		4.73	.017996
12			94		4.90	.049978
13	63	63	63		8.01	.059848
14			252		13.14	.049992
15	63	252			12.85	.058698
16		189			8.13	.049975
17	126				4.02	.050005
18		63			2.71	.049975
19		252			10.84	.049975
20	189				6.03	.049997
21		189	110		13.87	.049998
22	1371				43.73	.049996
23			63		3.29	.049989
24	149				4.75	.049997
25	4481	684	210		1796.61	.496938

TOTAL FLEET CAPABILITY.............. 4505.71

AVERAGE FLEET CAPABILITY............ 180.23

AVERAGE FLEET EFFICIENCY.................... .428126

Blue Fleet Capability and Efficiency

YEAR: 1992

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1			315		324.58	.999938
2			141		145.29	.999951
3			63		64.92	.999951
4			141		145.29	.999951
5			378		389.49	.999923
6			63		64.92	.999951
7		378			319.64	.999975
8		315			266.36	.999975
9		27			22.83	.999937
10		819	960		835.82	.496976
11			252		12.98	.049985
12			94		4.84	.049967
13	63	63	63		7.88	.060072
14			252		12.98	.049985
15			315		16.23	.050000
16		189			7.99	.049994
17		126			5.33	.050025
18		63			2.66	.049931
19		252			10.65	.049978
20	63	126			7.30	.067268
21		189	110		13.66	.050005
22	1639				51.37	.049996
23			63		3.25	.049992
24	149				4.67	.049999
25	4655	825	270		1935.11	.496911

TOTAL FLEET CAPABILITY.............. 4676.04

AVERAGE FLEET CAPABILITY............ 187.04

AVERAGE FLEET EFFICIENCY.................... .428825

Blue Fleet Capability and Efficiency

YEAR: 1993

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1			315		323.58	.999938
2			141		144.84	.999951
3			63		64.72	.999950
4			141		144.84	.999951
5			378		388.30	.999939
6			63		64.72	.999950
7			378		388.30	.999939
8		315			265.18	.999980
9		27			22.73	.999931
10		819	960		832.79	.496944
11			252		12.94	.049985
12			94		4.83	.050012
13	63	63	63		7.85	.061324
14			252		12.94	.049985
15			315		16.18	.050000
16			189		9.71	.050010
17		126			5.30	.049965
18		63			2.65	.049965
19		252			10.61	.050013
20		189			7.96	.050028
21		189	110		13.61	.050016
22	1772	56			57.64	.049997
23			63		3.24	.050062
24	149				4.65	.049996
25	4655	1203	342		2121.59	.517566

TOTAL FLEET CAPABILITY.............. 4931.70

AVERAGE FLEET CAPABILITY............ 197.27

AVERAGE FLEET EFFICIENCY.................... .429016

Blue Fleet Capability and Efficiency

YEAR: 1994

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1			315		313.66	.999976
2			141		140.40	.999966
3			63		62.73	.999951
4			141		140.40	.999966
5			378		376.39	.999978
6			63		62.73	.999951
7			378		376.39	.999978
8			315		313.66	.999976
9		27			21.72	.999885
10		819	960		802.50	.499406
11			252		12.55	.050014
12			94		4.68	.049999
13	63	63	63		7.55	.062621
14			252		12.55	.050014
15			315		15.68	.049990
16			189		9.41	.050001
17			126		6.27	.049974
18			63		3.14	.050054
19		252			10.13	.049970
20		189			7.60	.049987
21		63	236		14.28	.049987
22	1772	371			67.73	.050001
23			63		3.14	.050054
24	149				4.44	.049993
25	4466	1518	402		2128.67	.480892

TOTAL FLEET CAPABILITY.............. 4918.40

AVERAGE FLEET CAPABILITY............ 196.74

AVERAGE FLEET EFFICIENCY................... .427703

Blue Fleet Capability and Efficiency

YEAR: 1995

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1			315		297.79	.999966
2			141		133.29	.999958
3			63		59.56	.999961
4			141		133.29	.999958
5			378		357.34	.999974
6			63		59.56	.999961
7			378		357.34	.999974
8			315		297.79	.999966
9			27		25.52	.999776
10		630	1149		772.91	.496507
11			252		11.91	.049992
12			94		4.44	.049962
13	63	63	63		7.06	.083618
14			252		11.91	.049992
15			315		14.89	.050000
16			189		8.93	.049978
17			126		5.96	.050034
18			63		2.98	.050034
19			252		11.91	.049992
20		189			7.03	.039344
21			299		14.13	.049987
22	2020	749			83.55	.049988
23			63		2.98	.050034
24	149				4.11	.049998
25	4408	1734	438		2054.82	.496958

TOTAL FLEET CAPABILITY.............. 4741.00

AVERAGE FLEET CAPABILITY............ 189.64

AVERAGE FLEET EFFICIENCY.................... .428636

Blue Fleet Capability and Efficiency

YEAR: 1996

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1			315		282.90	.999941
2			141		126.63	.999972
3			63		56.58	.999954
4			141		126.63	.999972
5			378		339.48	.999938
6			63		56.58	.999954
7			378		339.48	.999938
8			315		282.90	.999941
9			27		24.25	.999941
10		252	1527		767.77	.559814
11			252		11.32	.050015
12			94		4.22	.049985
13	63	63	63		6.60	.081560
14			252		11.32	.050015
15			315		14.15	.050015
16			126	63	9.19	.093650
17			126		5.66	.050015
18			63		2.83	.050015
19			252		11.32	.050015
20			189		8.49	.050015
21			299		13.43	.050010
22	1768	1001			79.45	.088038
23			63		2.83	.050015
24	149				3.79	.050107
25	3691	2112	510		1884.01	.503447

TOTAL FLEET CAPABILITY............. 4471.81

AVERAGE FLEET CAPABILITY........... 178.87

AVERAGE FLEET EFFICIENCY.................... .435051

Blue Fleet Capability and Efficiency

YEAR: 1997

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1			315		266.71	.999994
2			141		119.38	.999962
3			63		53.34	.999962
4			141		119.38	.999962
5			378		320.05	.999994
6			63		53.34	.999962
7			378		320.05	.999994
8			315		266.71	.999994
9			27		22.86	.999962
10		252	1527		721.09	.557727
11			252		10.67	.050008
12			94		3.98	.050007
13	63	63	63		6.10	.080928
14			252		10.67	.050008
15			315		13.34	.050017
16				189	10.00	.091942
17			126		5.33	.049961
18			63		2.67	.049961
19			252		10.67	.050008
20			189		8.00	.049992
21			173	126	13.99	.091351
22	1254	1001	514		82.19	.094062
23			63		2.67	.049961
24	149				3.45	.050004
25	3691	2112	510		1722.72	.499688

TOTAL FLEET CAPABILITY.............. 4169.36

AVERAGE FLEET CAPABILITY............ 166.77

AVERAGE FLEET EFFICIENCY................... .436616

Blue Fleet Capability and Efficiency

YEAR: 1998

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1			315		259.09	.999897
2				141	144.93	.999932
3			63		51.82	.999936
4				141	144.93	.999932
5			378		310.91	.999904
6			63		51.82	.999936
7			378		310.91	.999904
8			315		259.09	.999897
9			27		22.21	.999936
10		111	1527	141	729.24	.520503
11			252		10.36	.049977
12				94	4.83	.050028
13	63	63	63		5.87	.081239
14			252		10.36	.049977
15			315		12.95	.049977
16				189	9.71	.050028
17			126		5.18	.049977
18			63		2.59	.049977
19			252		10.36	.049977
20			189		7.77	.049977
21			126	173	14.07	.049988
22	1081	1001	687		82.08	.089568
23			63		2.59	.049977
24	149				3.29	.050005
25	2971	2253	867		1676.78	.509061

TOTAL FLEET CAPABILITY.............. 4143.74

AVERAGE FLEET CAPABILITY............ 165.75

AVERAGE FLEET EFFICIENCY................... .431981

Blue Fleet Capability and Efficiency

YEAR: 1999

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1				315	321.71	.999976
2				141	144.00	.999976
3			63		51.48	.999922
4				141	144.00	.999976
5			378		308.90	.999987
6			63		51.48	.999922
7			378		308.90	.999987
8			315		257.42	.999987
9			27		22.06	.999793
10			1512	267	749.62	.496995
11			252		10.30	.050016
12				94	4.80	.049999
13	63	63		63	6.46	.100401
14			252		10.30	.050016
15			315		12.87	.049996
16				189	9.65	.049999
17			126		5.15	.050016
18			63		2.57	.050016
19			252		10.30	.050016
20			189		7.72	.049983
21			126	173	13.98	.049990
22	1081	1001	687		81.33	.144865
23				63	3.22	.050045
24			149		6.09	.050015
25	2251	2385	1197	135	1745.51	.515624

TOTAL FLEET CAPABILITY.............. 4289.82

AVERAGE FLEET CAPABILITY............ 171.59

AVERAGE FLEET EFFICIENCY.................... .434301

Blue Fleet Capability and Efficiency

YEAR: 2000

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1				315	317.55	.999990
2				141	142.14	.999990
3			63		50.82	1.000000
4				141	142.14	.999990
5			378		304.89	.999902
6			63		50.82	1.000000
7			378		304.89	.999902
8			315		254.08	.999921
9			27		21.78	1.000000
10			1197	582	771.45	.496975
11				252	12.70	.049992
12				94	4.74	.050021
13	63		63	63	7.07	.061839
14			252		10.16	.049980
15			315		12.70	.049980
16				189	9.53	.050018
17			126		5.08	.049980
18			63		2.54	.049980
19			252		10.16	.049980
20			189		7.62	.049980
21			63	236	14.44	.050012
22	892	1001	876		83.42	.118052
23				63	3.18	.050018
24			149		6.01	.050003
25	1531	2385	1512	195	1716.20	.519419

TOTAL FLEET CAPABILITY.............. 4266.11

AVERAGE FLEET CAPABILITY............ 170.64

AVERAGE FLEET EFFICIENCY.................... .431837

* CUMULATIVE AVERAGE FLEET EFFICIENCY....... .430273

* The cumulative average fleet efficiency represents the mean of the yearly average fleet efficiency measures for the period 1987 to 2000.

Example Linear Programming Problem

Format to Determine the Relative Unit Efficiency of Unit 14

Minimize: 0 X1 + 0 X2 + 252 X3 + 0 X4

Subject to:

Row 1:	272.03 X1	-	0 X2	-	315 X3	-	0 X4	<=	0
Row 2:	121.77 X1	-	0 X2	-	141 X3	-	0 X4	<=	0
Row 3:	54.41 X1	-	0 X2	-	63 X3	-	0 X4	<=	0
Row 4:	326.44 X1	-	0 X2	-	378 X3	-	0 X4	<=	0
Row 5:	40.35 X1	-	63 X2	-	0 X3	-	0 X4	<=	0
Row 6:	242.11 X1	-	378 X2	-	0 X3	-	0 X4	<=	0
Row 7:	201.76 X1	-	315 X2	-	0 X3	-	0 X4	<=	0
Row 8:	17.29 X1	-	27 X2	-	0 X3	-	0 X4	<=	0
Row 9:	672.75 X1	-	819 X2	-	960 X3	-	0 X4	<=	0
Row 10:	13.17 X1	-	0 X2	-	0 X3	-	252 X4	<=	0
Row 11:	4.06 X1	-	0 X2	-	94 X3	-	0 X4	<=	0
Row 12:	2.72 X1	-	0 X2	-	63 X3	-	0 X4	<=	0
Row 13:	10.09 X1	-	315 X2	-	0 X3	-	0 X4	<=	0
Row 14:	4.04 X1	-	126 X2	-	0 X3	-	0 X4	<=	0
Row 15:	6.05 X1	-	189 X2	-	0 X3	-	0 X4	<=	0
Row 16:	11.68 X1	-	110 X2	-	189 X3	-	0 X4	<=	0
Row 17:	44.71 X1	-	1396 X2	-	0 X3	-	0 X4	<=	0
Row 18:	3.97 X1	-	124 X2	-	0 X3	-	0 X4	<=	0
Row 19:	1129.66 X1	-	3035 X2	-	381 X3	-	0 X4	<=	0
Row 20:	10.88 X1	-	0 X2	-	0 X3	-	0 X4	=	1

where:

X1 = output multiplier U1 (Capability)

X2 = input multiplier V1 (Blue 1)

X3 = input multiplier V2 (Blue 2)

X4 = input multiplier V3 (Blue 3)

Output Summary for Unit Number 14

THE SOLUTION FOR PROBLEM   TANK DATA

```
X( 1  ) =   9.191176E-02
X( 2  ) =   5.887022E-02
X( 3  ) =   7.937967E-02
X( 4  ) =   4.803484E-03
S( 1  ) =   1.838235E-03
S( 2  ) =   4.376751E-04
S( 4  ) =   1.838235E-03
S( 5  ) =   1.838235E-04
S( 6  ) =   1.838235E-04
S( 8  ) =   3.413866E-04
S( 9  ) =   62.58555
S( 11  ) =   7.088527
S( 12  ) =   4.750919
S( 13  ) =   17.61673
S( 14  ) =   7.046323
S( 15  ) =   10.5704
S( 16  ) =   20.40495
S( 17  ) =   78.07345
S( 18  ) =   6.935017
S( 19  ) =   105.0857
```

Solution to the Dual

Row	Dual Value
1	0
2	0
3	-4
4	0
5	0
6	0
7	-7.445117E-15
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0
16	0
17	0
18	0
19	0
20	20.00368

The minimum value of the objective function is :   20.00368

Output Summary for Unit Number 14

Sensitivity to the Right Hand Side:

Row Number	Variable Out	Lower Bound	Upper Bound
1	S(1)	-1.838E-03	INFINITY
2	S(2)	-4.376E-04	INFINITY
3	NONE	INFINITY	1.955E-04
4	S(4)	-1.838E-03	INFINITY
5	S(5)	-1.838E-04	INFINITY
6	S(6)	-1.838E-04	INFINITY
7	NONE	INFINITY	1.531E-04
8	S(8)	-3.413E-04	INFINITY
9	S(9)	-62.585	INFINITY
10	NONE	INFINITY	1.210
11	S(11)	-7.088	INFINITY
12	S(12)	-4.750	INFINITY
13	S(13)	-17.616	INFINITY
14	S(14)	-7.046	INFINITY
15	S(15)	-10.570	INFINITY
16	S(16)	-20.404	INFINITY
17	S(17)	-78.073	INFINITY
18	S(18)	-6.935	INFINITY
19	S(19)	-105.085	INFINITY
20	S(4)	1E-09	INFINITY

Range of Cost Coefficients of Non-Basic Variables:

Variable	Variable Out	Lower Bound
S(3)	X(0)	-4
S(7)	X(0)	-1.000E-08
S(10)	X(0)	-1E-08

Output Summary for Unit Number 14

Range of Costs of Basic Variables:

Variable	Variable In	Lower Bound	Upper Bound
X(1)	NONE	INFINITY	INFINITY
X(2)	S(7)	-1.000E-08	INFINITY
X(3)	S(3)	-1E-08	INFINITY
X(4)	S(10)	-1E-08	INFINITY
S(1)	S(3)	-.8	INFINITY
S(2)	S(3)	-1.787	INFINITY
S(4)	S(3)	-.666	INFINITY
S(5)	S(7)	-1.000E-08	INFINITY
S(6)	S(7)	-1.000E-08	INFINITY
S(8)	S(7)	-1.000E-08	INFINITY
S(9)	S(7)	-1E-08	INFINITY
S(11)	S(3)	-2.680	INFINITY
S(12)	S(3)	-4	INFINITY
S(13)	S(7)	-1.000E-08	INFINITY
S(14)	S(7)	-1.000E-08	INFINITY
S(15)	S(7)	-1.000E-08	INFINITY
S(16)	S(7)	-1.000E-08	INFINITY
S(17)	S(7)	-1E-08	INFINITY
S(18)	S(7)	-1.000E-08	INFINITY
S(19)	S(7)	-1E-08	INFINITY

Blue Fleet Capability and Efficiency

Sensitivity to Deployment Time Modification *

YEAR: 1987

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1		315			272.03	.999926
2		141			121.77	.999961
3		63			54.41	.999961
4		141			121.77	.999961
5		378			326.44	.999939
6	63				40.35	.999950
7	378				242.11	.999992
8	315				201.76	.999992
9	27				17.29	.999785
10	819	960			849.26	.627371
11			252		35.73	.135657
12		94			11.01	.135619
13		63			7.38	.135637
14		252			29.51	.135637
15	315				27.36	.135607
16		189			22.13	.135619
17	126				10.94	.135557
18	63				5.47	.135557
19		252			29.51	.135637
20	189				16.41	.135557
21	110	189			31.69	.135609
22	1396				121.25	.135604
23	63				5.47	.135557
24	124				10.77	.135603
25	3035	381			1426.05	.627389

TOTAL FLEET CAPABILITY.............. 4037.87

AVERAGE FLEET CAPABILITY............ 161.51

AVERAGE FLEET EFFICIENCY................... .486107

* For this analysis, the delivery time for POMCUS units is changed from 21 days to 14 days; the delivery time for CONUS units is changed from 90 days to 60 days.

Sample Calculation for the Capability of Unit 14

Capability = Z * Aijk * Rj * Bi * Mk (t = 1)

Capability = (0.1356)*(1.75)*(0.81)*(252)*(0.50) +

(0.1356)*(1.63)*(0.19)*(252)*(0.50)

Capability = 29.51

Blue Fleet Capability and Efficiency

Sensitivity to Deployment Time and Mission Profile Modifications *

YEAR: 1987

UNIT	BLUE1	BLUE2	BLUE3	BLUE4	CAPABILITY	EFFICIENCY
1		315			272.03	.999966
2		141			121.77	.999966
3		63			54.41	.999961
4		141			121.77	.999966
5		378			326.44	.999961
6	63				40.35	.999950
7	378				242.11	.999992
8	315				201.76	.999992
9	27				17.29	.999992
10	819	960			1273.90	.941064
11			252		53.59	.203335
12		94			16.51	.203367
13		63			11.07	.203455
14		252			44.27	.203455
15	315				41.04	.203410
16		189			33.20	.203394
17	126				16.41	.203336
18	63				8.21	.203336
19		252			44.27	.203455
20	189				24.62	.203377
21	110	189			47.53	.203393
22	1396				181.87	.203400
23	63				8.21	.203336
24	124				16.15	.203342
25	3035	381			2139.07	.941081

TOTAL FLEET CAPABILITY.............. 5357.85

AVERAGE FLEET CAPABILITY............ 214.31

AVERAGE FLEET EFFICIENCY.................... .549171

* For this analysis, in addition to the modifications made in Appendix E, the mission profile of percent time in the defense is changed from 0.50 to 0.75 for all POMCUS and CONUS units.

Sample Calculation for the Capability of Unit 14

$$\text{Capability} = Z * A_{ijk} * R_j * B_i * M_k \qquad (t = 1)$$

$$\text{Capability} = (0.1356)*(1.75)*(0.81)*(252)*(0.80) + (0.1356)*(1.63)*(0.19)*(252)*(0.80)$$

$$\text{Capability} = 44.27$$

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Vita

Captain Michael S. Remias was born on 18 July 1954 in Bethesda, Maryland. He graduated from high school in Cincinnati, Ohio in 1972 where he received appointment to the United States Military Academy. Upon graduation from the United States Military Academy in June 1976, Captain Remias was commissioned as an officer of Field Artillery. His military education includes completion of the United States Army Airborne and Ranger Schools, the Field Artillery Officer Basic, Cannon, and Advanced Courses, and the Defense Language Institute Greek Language Course.

Captain Remias has experienced a broad spectrum of field artillery assignments. He has served as a Fire Support Officer, Fire Direction Officer, and Executive Officer in the First Cavalry Division, Fort Hood, Texas. He served as a Special Weapons Team Officer with the Southern European Task Force, Greece. Serving as a Senior Gunnery Instructor with the Field Artillery School, Captain Remias accepted command as a Battery Commander in the Field Artillery Training Center Fort Sill, Oklahoma in November 1983. He successfully completed his command tour in May 1985 and entered the School of Engineering, Air Force Institute of Technology, in June of the same year.

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The primary objective of this thesis research was to determine the relative unit efficiencies of the expected attrition capability of United States armor forces. A measure of effectiveness defining the unit attrition capability was derived using the Lanchester-type equations of heterogeneous combat as a functional basis. Included in the armor effectiveness measure was a parameter describing the unit's discounted time value of the attrition process. The discounted time value of a unit is characterized by exponential decay reflecting the situationally dependent value of the unit to influence the battle engagement.

The determination and analysis of the relative unit efficiencies was accomplished using the data envelopment model fo Charnes, Cooper, and Rhodes. The non-linear relation characteristic of the efficiency equation reduces to a linear programming problem by the method of linear fractional programming. From this analysis and methodology, important decisions regarding the efficient deployment and employment of armor assets can be quantitatively assessed. The results of this thesis research indicate that the analytical methodology contained in this thesis can be used as a method for the comparison of armor procurement, production and employment options.

END

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